Determination of Flaw Size and Depth from Temporal Evolution of Thermal Response

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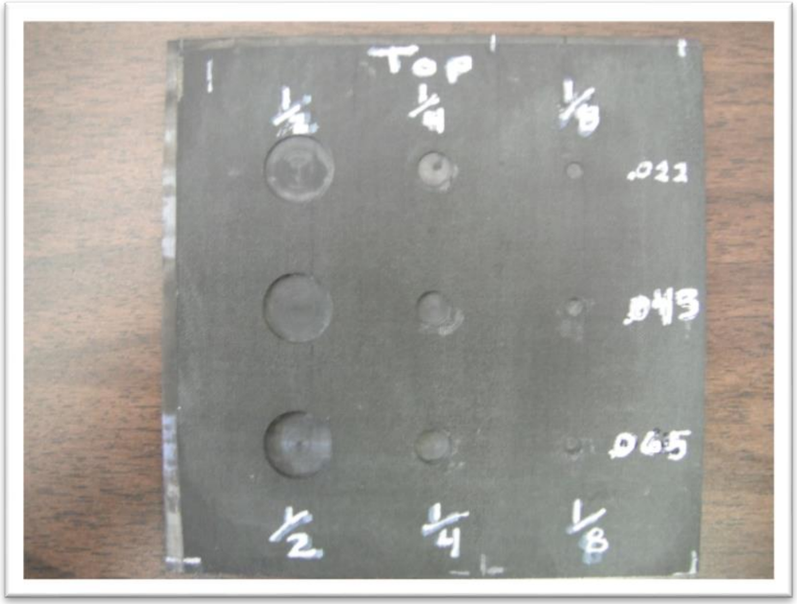
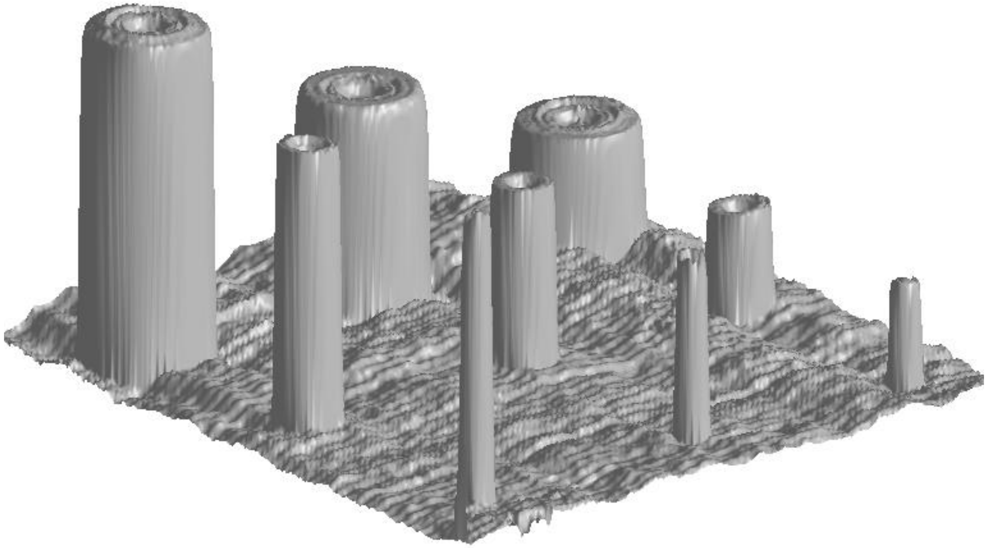
Outline

- Difficulty of sizing flaws with thermography
- Variational Principle in Image Processing
- Approximate solution for flat bottom holes
- Application of variational principle to hole sizing with multiple time images
- Discussion of limitations
- Summary

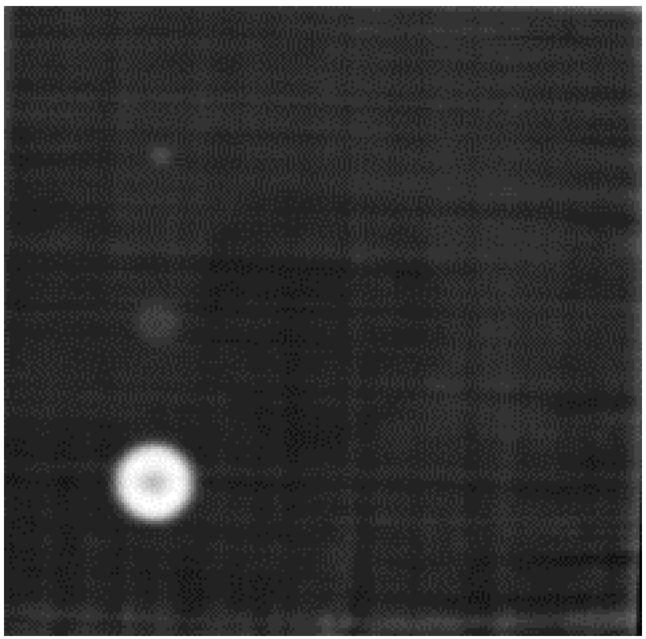
Composite "Flat" Bottom Hole Specimen

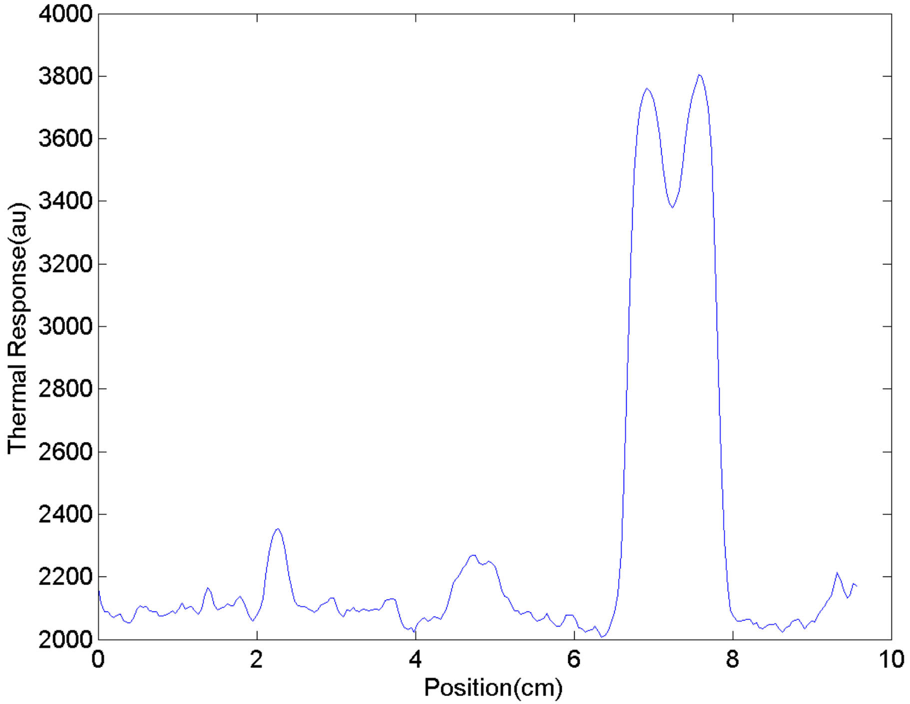




Approximate Depths of Flat Bottom Holes 0.05, 0.1, 0.15 cm
Approximate Diameters 1.27, 0.63, 0.32 cm

Thermal Response at 0.25 seconds





Profile across center of flat bottom holes

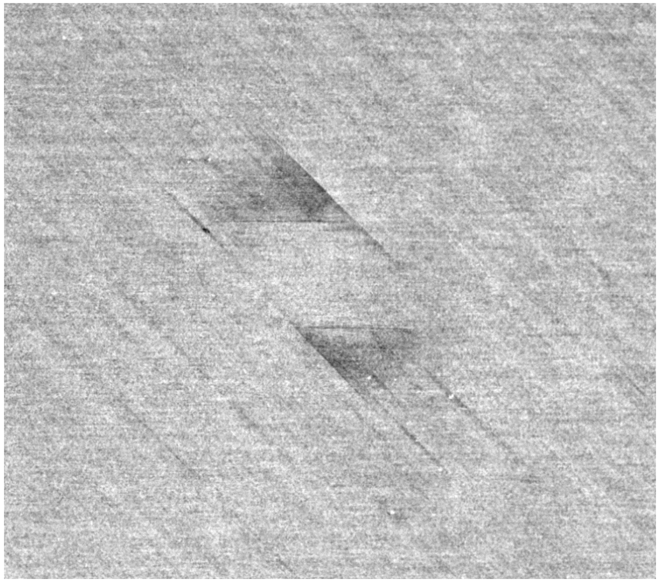
Thermographic Sizing of Flaws

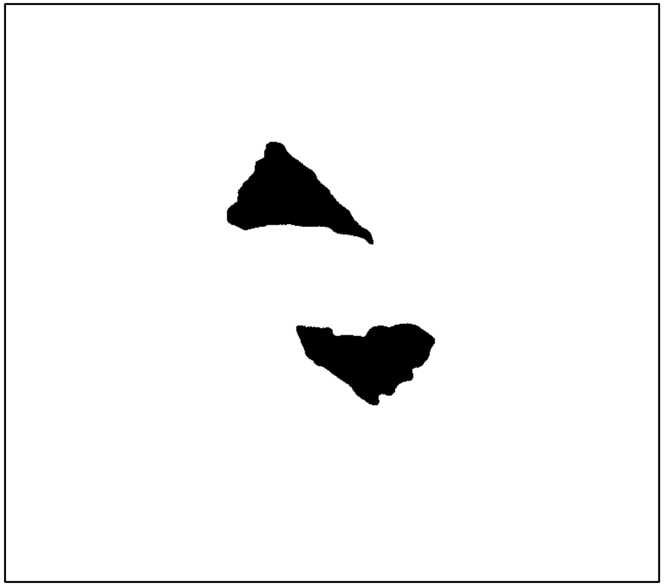
- Size of small flaws complicated by in-plane thermal diffusion
- Two common approaches have been taken
  - Point by point time analysis – flaw sizing from earliest flaw response (limited in-plane diffusion)
  - Image analysis- deconvolution or other image processing technique to remove diffusion effects
- Alternate approach is simultaneously incorporate time and spatial information into analysis
  - Simultaneous application of variational method for multiple time responses

Variational Principle in Image Processing

- Applications:
  - De-noising
  - Segmentation
  - Deblurring
  - Estimation of apparent motion
  - Registration
- Minimize energy $\Rightarrow E(u) = \int_{\Omega} F(u, I)dx\, dy + \gamma\, \mathrm{G(u)}$
  - *u – desired image*
  - *I – acquired image*
  - *E(u) – Energy*
  - *F(u,I) – relationship between desired image and acquired image*
  - *G(u)- constraint*
  - γ *- constant*
  - Ω *- domain*

Variational Principle in Image Segmentation





Chan, T. F. and Vese, L. A., “Active contours without edges," IEEE Transactions on Image Processing 10, 266-277 (2001)

Variational Principle for Sizing Thermographic Flaws Using Single Image

- Minimize energy $\Rightarrow E(u) =$
  $\int_{\Omega} (D(u(x,y),x,y,t) - T(x,y,t))^2 dx\,dy + \gamma \int_{\Omega} |\nabla u|\,dx\,dy$
  - $D(x,y,t)$ – *flaw response for flaw with shape u*
  - *u(x,y) – binary representation of hole (0 not hole, 1 hole)*
  - $T(x,y,t)$ – *measured response*
  - *E(u) – Energy*
  - $\int_{\Omega} |\nabla u|\,dx\,dy$– *circumference of flaw*
- Need fast method for estimating
  - $D(u(x,y),x,y,t)$

Approximate Solution for Flat Bottom Hole of the Form

$$D(u,x,y,t) = \frac{2F\sqrt{\kappa}e^{-d^2/(\kappa t)}}{K\sqrt{\pi t}} \frac{1}{\pi b t} \iint_S u(x,y) e^{-((x-x')^2+(y-y')^2)/(bt)} dx' dy'$$

D(u,x,y,t) – Thermal response of flat bottom hole

d – depth of top of flat bottom hole

S – surface of top of flat bottom hole

κ- thermal diffusivity of material

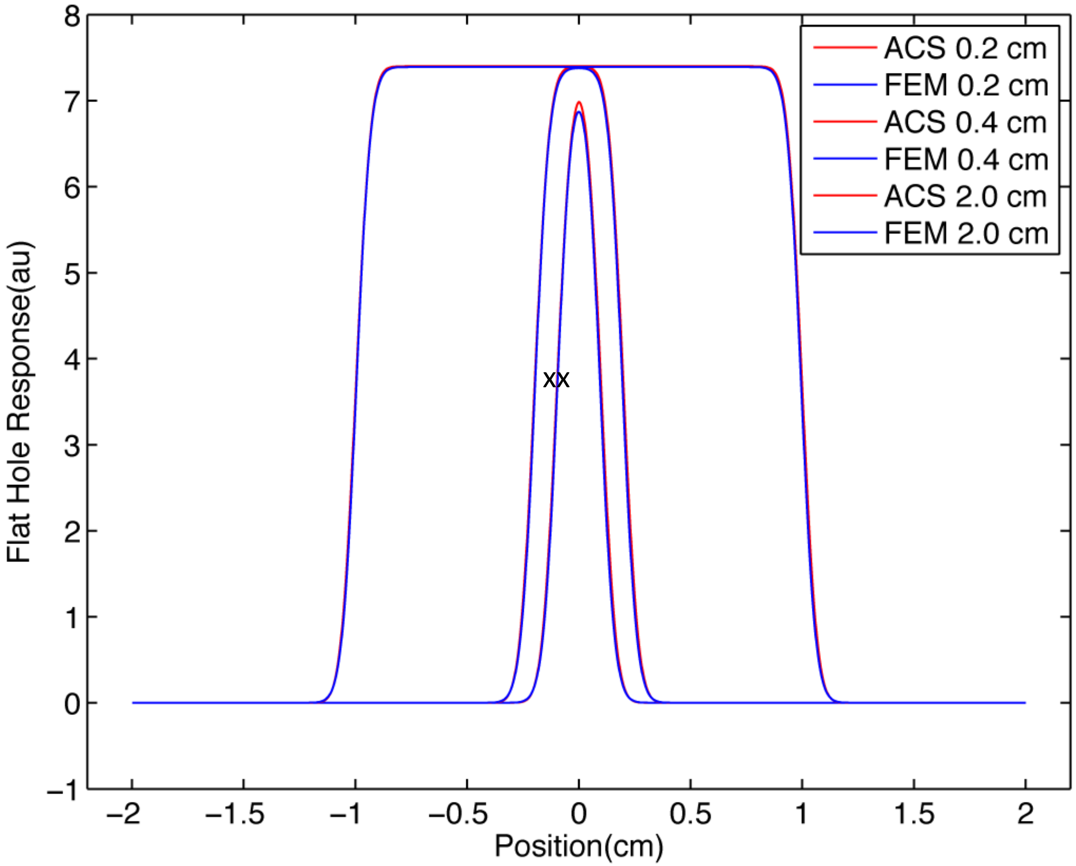
K-thermal conductivity

F-flux at surface

b-constant

Note: Blurring of the delamination is function of the time of measurement, not the depth of the top of the flat bottom hole

Thermal Response Slot 0.1 cm Below Surface at 0.5 Seconds

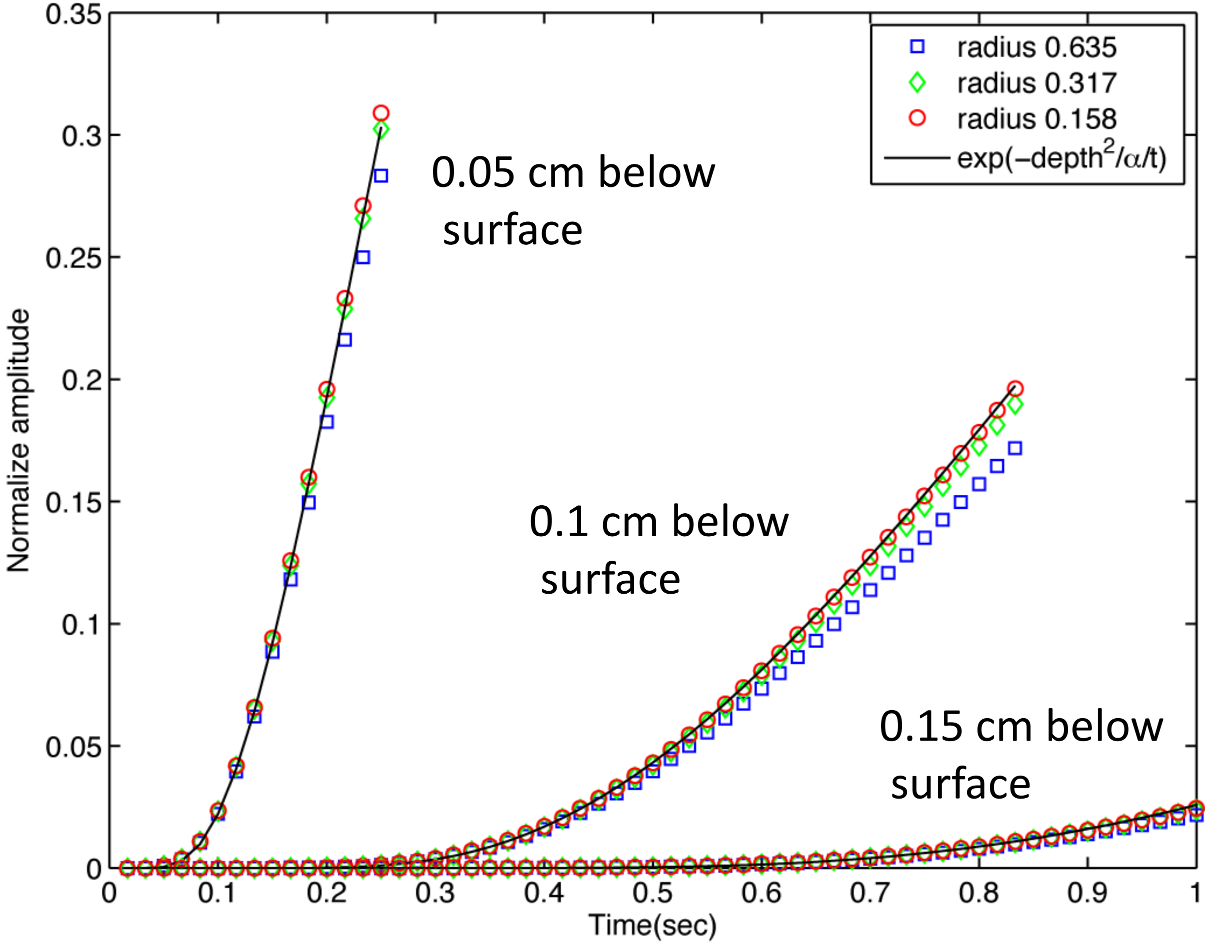


ACS – Approximate Convolution Solution with b=2.1*in-plane diffusivity

FEM – Finite Element Simulations

Both size and shape in good agreement

Amplitude of Thermal Response for Different Hole Sizes and Depths



Time dependence of amplitude in good agreement with approximate solution

Single Time Slice Analysis

- Euler-Lagrange Equation of single time response

$$\delta(\phi(x,y))(G * (G * H(\phi(x,y))\text{-T(x,y,t)} + \gamma\, div(\frac{\nabla\phi(x,y)}{|\nabla\phi(x,y)|})=\frac{d(\phi(x,y))}{dt}$$

  - G- Gaussian convolution
  - H – Approximate Hat Function
  - δ – Approximate delta function
  - T(x,y,t) – measured flaw response
  - Solving for $\frac{d(\phi(x,y))}{dt}$=0

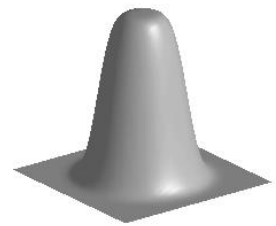
Details in paper

Relationship of $\phi(x, y)$ to Thermal Response

$\phi(x, y)$

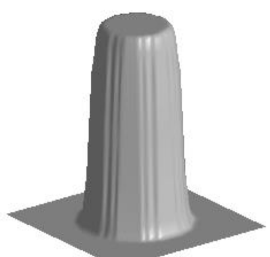
Thermal response of flat bottom hole

$G * H(\phi(x, y))$

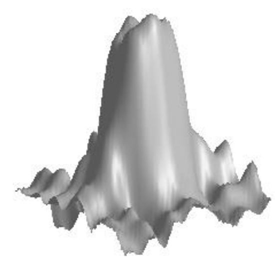


Shape of flat bottom hole

$H(\phi(x, y))$

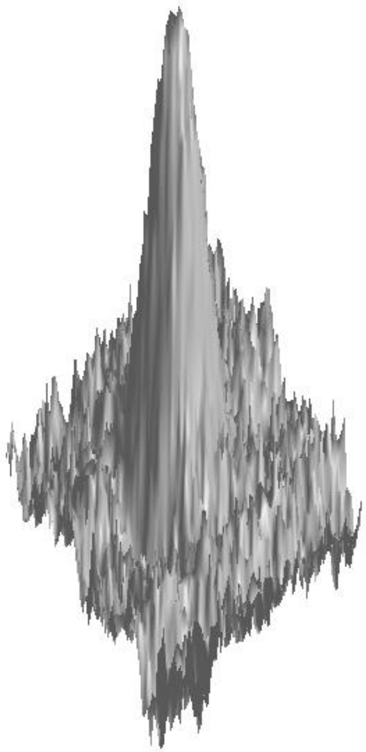


Measured Thermal Response

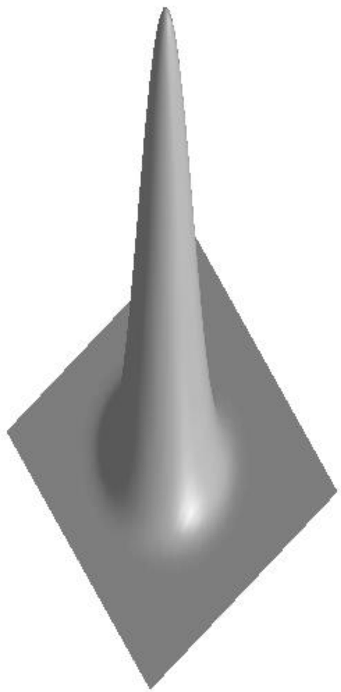


Difference between $G * H\big(\phi(x, y)\big)$ and Measured Thermal Response Determines $\frac{d(\phi(x,y))}{dt}$

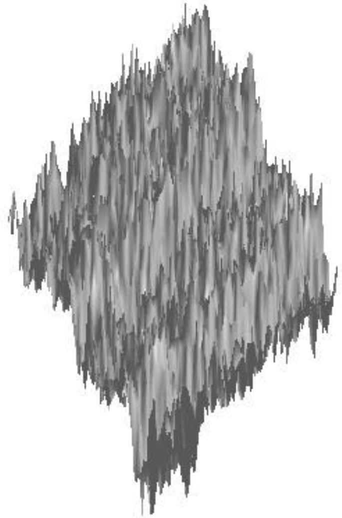
Results for 0.635cm Diameter Flat Bottom Hole at 1 sec



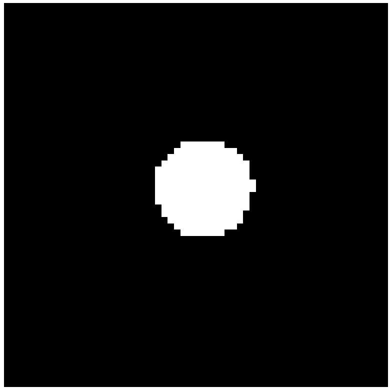
Measured Response



Estimated Response



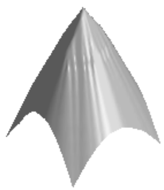
Difference



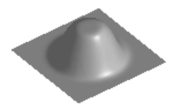
Estimated Image of Top of Flat Bottom Hole (estimated diameter 0.64)

From Single $\phi(x, y)$ to Thermal Response at Multiple Times

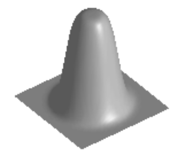
$\phi(x, y)$



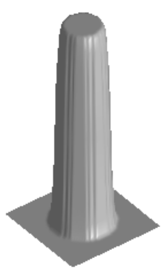
Thermal response at 0.5 sec



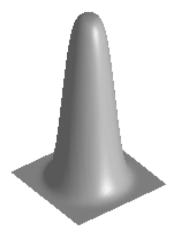
Thermal response at 0.9 sec



Shape of flat bottom hole $H(\phi(x, y))$



Thermal response at 0.7 sec



Time Series Analysis

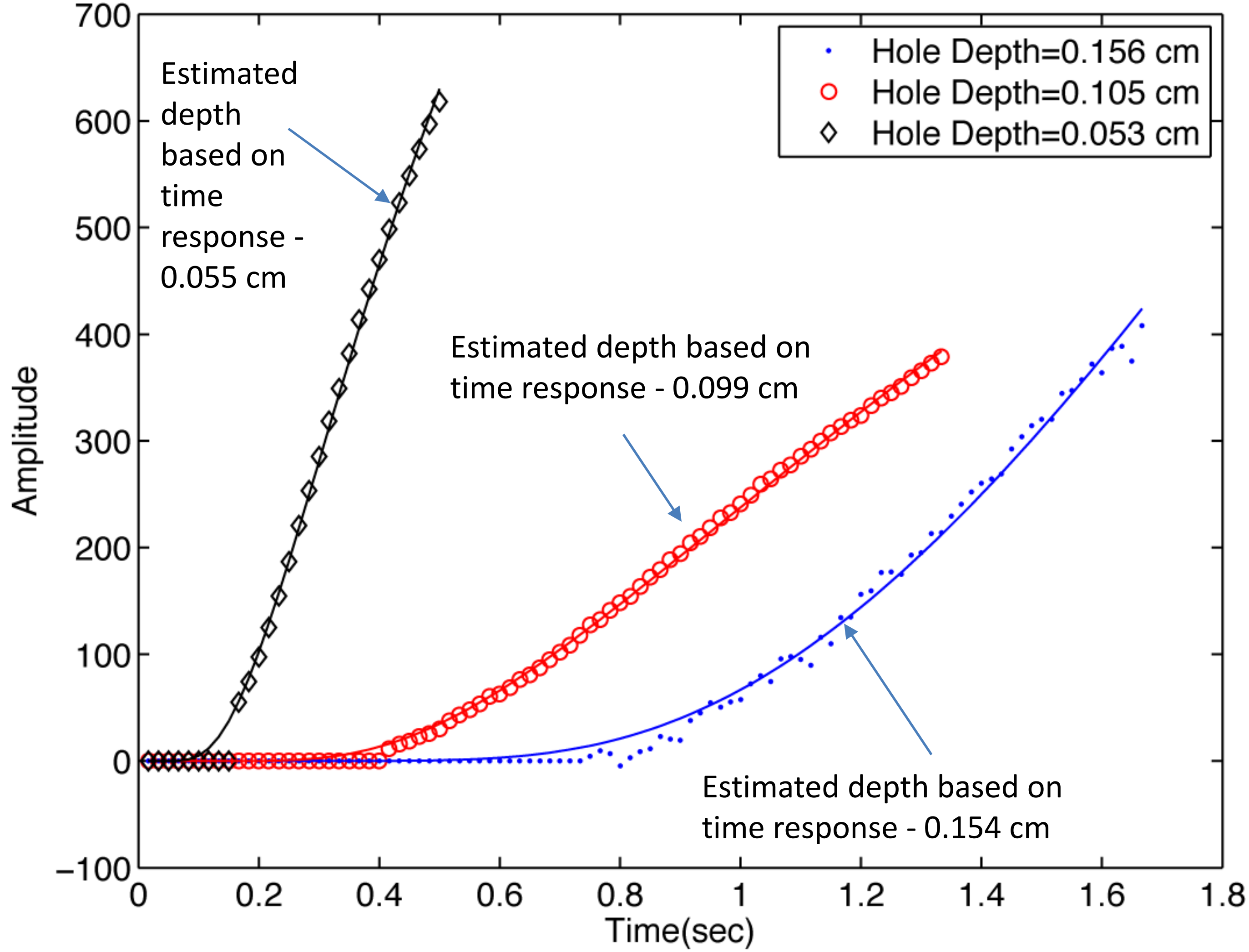
- Euler-Lagrange Equation of a series of time response

$$\delta(\phi(x,y))\left(\sum_n (G * (G * H(\phi(x,y)) - T(x,y,t_n)\right) + \gamma \, div\left(\frac{\nabla\phi(x,y)}{|\nabla\phi(x,y)|}\right) = \frac{d(\phi(x,y))}{dt}$$

  - $T(x,y,t_n)$ – measured flaw response at t_n
  - Solving for $\frac{d(\phi(x,y))}{dt}=0$

Details in paper

Time Response of Amplitude of Thermal Response



Results on All Holes

Depth(cm)	Time Series Image Analysis Depth Estimate (cm)	Diameter(cm)	Average Single Image Analysis Diameter (cm)	Time Series Image Analysis Diameter (cm)
0.03	0.04	1.27	1.19	1.22
0.10	0.09	1.27	1.23	1.22
0.15	0.14	1.27	1.23	1.19
0.05	0.05	0.63	0.62	0.62
0.10	0.10	0.63	0.63	0.63
0.15	0.15	0.63	0.65	0.64
0.05	0.05	0.32	0.34	0.30
0.10	0.10	0.32	0.36	0.30
0.15	0.14	0.32	0.56	0.16

Discussion

- Most significant improvement is on smallest holes
- Estimation of flaw size depth incorporates flaw size information and give good agreement with known depths for all flaws
  - Earliest significant responses give best estimates for depth
- Constraint minimizing circumference of flaw tends to result in circular flaws
  - Good for this case, however, not so good if flaws are not circular
- More work need for delaminations
  - Single delaminations
  - Overlapping delaminations

Summary

- Good approximate solution for thermal response of flat bottom holes is the convolution of the shape of the top of the flat bottom hole with a Gaussian
- Variational method for determining the size and depth of flat bottom holes gives a more accurate value for the size and depth